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A multisector perspective on wage stagnation [☆]

L. Rachel Ngai [a,b,c,d,*], Orhun Sevinc [e,b]

[a] *London School of Economics, United Kingdom of Great Britain and Northern Ireland*
[b] *CFM, United Kingdom of Great Britain and Northern Ireland*
[c] *CEPR, United Kingdom of Great Britain and Northern Ireland*
[d] *IZA, Germany*
[e] *Central Bank of the Republic of Türkiye, Türkiye*

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ABSTRACT

Low-skill workers are concentrated in sectors experiencing fast productivity growth, yet their real wages have stagnated and lagged behind aggregate productivity. We provide evidence demonstrating the importance of a multisector perspective. Central to our mechanism is the decline in the relative price of the low-skill intensive sector driven by its faster productivity growth. This dampens wage gains for low-skill workers by lowering the price of their output relative to their consumption basket, which is further reinforced by shifting them into the sector where less weight is placed on their labor. We calibrate the two-sector model to the 1980–2010 U.S. economy and find this mechanism to be quantitatively important. Our counterfactual analysis reveals that low-skill real wage growth would have nearly doubled if the observed aggregate productivity growth had been evenly distributed across sectors.

1. Introduction

Low-skill workers have experienced very little wage growth, despite working mostly in sectors with fast productivity growth. In the U.S., the real wage of non-college workers increased by about 20% between 1980–2010, which is less than half the increase in aggregate labor productivity.[1] The low-skill wage "stagnation" persists even after controlling for age, race, gender, education, and occupation, indicating it is not due to compositional changes in low-skill employment.[2] Hours worked by these workers represent two-thirds of overall hours worked, so their wage stagnation explains why the average wage is lagging behind aggregate labor productivity,

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* Corresponding author.

E-mail addresses: L.Ngai@lse.ac.uk (L.R. Ngai), Orhun.Sevinc@tcmb.gov.tr (O. Sevinc).

[1] The precise increase in the aggregate non-college real wage range from 15% to 25%, depending on the choice of price deflators, composition adjustment, the inclusion of non-wage compensation and self-employment, and whether it is only for the nonfarm business sectors. Regardless of these choices, the finding that the non-college real wage has had little growth and lags behind the aggregate labor productivity growth is robust.

[2] As documented in Acemoglu and Autor (2011), low-skill wage stagnation coexists with occupational polarization (low-wage occupations have faster wage growth than middle-wage occupations). The low-skill wage stagnation pertains to a group of workers with given education qualifications, whereas polarization is defined over given occupational groups irrespective of who is employed there. Sevinc (2019) documents the role of skill heterogeneity within an occupation in understanding these two patterns.

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despite the real wage of college graduates growing faster than aggregate labor productivity. Taken together, these observations reject the view that a rising tide lifts all boats; apparently, many boats are left behind.

Our objective is to understand why the growth of the low-skill real wage has been so low and lagging behind aggregate labor productivity. We offer a novel multisector perspective, where the key mechanism is the falling relative price driven by faster productivity growth in sectors that use low-skill workers more intensively. This mechanism dampens the positive effect of productivity on the low-skill real wage, which is the average *value* of the marginal product of low-skill workers, through two channels. First, the increase in the physical marginal output caused by faster productivity growth is *valued* at a lower price relative to their consumption basket. Second, when outputs are complements across sectors, this leads to a reallocation of low-skill workers to the high-skill intensive sector where they have a lower weight in the production function.

We provide motivating evidence from the U.S. to support this multisector mechanism. The low-skill real wage was growing at about the same rate as the high-skill real wage and the aggregate labor productivity before 1980 before it started to lag behind both series for the next three decades. We classify sectors into high-skill intensive sector and low-skill intensive sector according to their high-skill labor income shares. Interestingly, we find that the rise in the relative productivity of the low-skill intensive sector also started around 1980, and this is mirrored by the fall in the relative price of the low-skill sector. The reallocation of low-skill workers into the high-skill intensive sector also took off during this period.

Using the accounting identity that the total value-added of the economy equals the sum of total factor payments, we show that capital plays an essential role in understanding the divergence between the low-skill wage and aggregate labor productivity. This identity reveals that there are three driving forces behind the divergence: the increasing skill premium, the declining labor income share, and the rising relative cost of living, measured by the ratio of the consumption deflator to the output deflator. The latter two forces, which together account for 30% to 50% of the divergence, require the presence of capital. In its absence, both the labor income share and the relative price of consumption would equal one.

To quantify the proposed mechanism, we calibrate a two-sector model to match key features of the US economy from 1980 to 2010. Production in both sectors uses low-skill labor, high-skill labor, and capital. The low-skill sector uses low-skill labor more intensively and has faster productivity growth. As in Buera et al. (2022), we show that the faster productivity growth in the low-skill sector leads to an increase in the skill premium, which contributes to the divergence. In addition, due to the presence of capital in our model, we find that the multisector mechanism also contributes to the divergence by increasing the relative cost of living.

In addition to our mechanism through uneven productivity growth, the calibration also allows for four other forces that are shown to be important for understanding the skill premium and the labor share. They are the falling relative price of capital in the presence of capital-skill complementarity (Krusell et al., 2000), the falling production weights of low-skill labor (Goldin and Katz, 2009), and the skill-biased demand and supply shifts (Katz and Murphy, 1992).

The uneven productivity growth, which is calibrated to match the observed changes in relative prices, is quantitatively important for both the divergence and low-skill wage stagnation. This can be demonstrated by considering what would happen to the low-skill wage if instead the same level of aggregate productivity growth were driven by a balanced increase in sectoral productivity. The result of this counterfactual analysis is that the increase in the low-skill wage would have been almost double, and the resulting divergence would have been nearly halved. This highlights that the source of aggregate productivity growth is crucial for understanding low-skill wage stagnation.

The declining production weights of low-skill labor also play an important role, as they are a key factor driving the decrease in the labor share and the increase in the skill premium. Their contribution to the stagnation of low-skill wage relies on lowering the marginal product of low-skill labor in both sectors, which fails to account for the observed differential trends. These differential trends are a result of changing relative prices when the growth of nominal low-skill wages is similar across sectors. Both the decline in the relative price of capital and the skill-biased demand shifts that increase the production weight of high-skill labor are quantitatively important for the rise in the skill premium but not for low-skill wage stagnation. These quantitative exercises demonstrate that factors contributing to the increase in the skill premium do not necessarily contribute to low-skill wage stagnation.

Our paper can be viewed as providing a framework for assessing the quantitative significance of various forces underlying key aspects of labor market inequalities and their roles in understanding low-skill wage stagnation. Since the seminal work of Katz and Murphy (1992), an extensive literature has emerged studying the effects of skill-biased demand and supply shifts on the skill premium, with a particular focus on skill-biased technical change (see Goldin and Katz, 2009, for a review). However, skill-biased technical change that simply improves the relative productivity of high-skill workers does not necessarily contribute to stagnation in low-skill wage (Johnson, 1997; Acemoglu and Autor, 2011). This limitation has partly contributed to a growing literature on automation and declining labor shares (see Zeira, 1998; Karabarbounis and Neiman, 2014; Acemoglu and Restrepo, 2018; Martinez, 2019; Caselli and Manning, 2019; Hémous and Olsen, 2022; Moll et al., 2022; Hubmer, 2023, among others).[3] Other potential explanations include de-unionization and the decline in the minimum wage (Lee, 1999; Dustmann et al., 2009), increasing monopsony power (Manning, 2003), rising imports (Autor et al., 2013), and the decline in the urban premium for non-college workers (Autor, 2019).[4] Many of these forces can be understood within the conceptual framework of one-sector models. Our contribution to this literature is to emphasize the importance of sector-specific technological changes.

[3] This is accompanied by a parallel growing empirical literature on the effect of automation on employment, wages and labor income shares (see Autor and Salomons, 2018; Graetz and Michaels, 2018; Acemoglu and Restrepo, 2020; Chen et al., 2021; Kapetaniou and Pissarides, 2020, among others).

[4] To the extent that most of the expansion in high-skill services occurs in urban areas, our mechanism is consistent with the finding of Autor (2019) on the decline of the urban premium for non-college workers due to region-specific occupational changes.

In exploring the role of uneven productivity growth on the labor market by skill groups, Buera et al. (2022) is the closest work to ours in terms of explaining the rise in the skill premium and the expansion of the high-skill intensive sector.[5] The main contributions of our paper, relative to theirs, are to demonstrate the effects of uneven productivity growth on low-skill wage growth and to elucidate the roles of changing relative prices and sectoral reallocation of labor in driving these outcomes. In addition, capital, absent from their model, plays two crucial roles in our analysis. First, its effect on the labor share and the relative price of consumption is essential to study the decoupling of wages and aggregate productivity. Second, it provides an additional mechanism for the increase in the skill premium through capital-skill complementarity and a decreasing relative price of capital, as in Krusell et al. (2000).

Section 2 presents motivating facts on understanding low-skill wage stagnation and the importance of a multisector perspective. Section 3 introduces a two-sector model with three factors of production: low-skill labor, high-skill labor, and capital. Section 4 calibrates the model to assess the quantitative significance of the multi-sector mechanism alongside other forces. Section 5 concludes.

2. Motivation

This section presents a set of motivating facts for understanding low-skill real wage in a multisector economy. We focus on documenting facts related to its stagnation and divergence from aggregate labor productivity.

2.1. Data

The primary data sources used in the paper are the March 2017 release of the World KLEMS (Jorgenson et al., 2017) assisted by the April 2013 Release (Jorgenson et al., 2012) and the Current Population Survey (CPS) sourced from IPUMS (Flood et al., 2020).

The aggregate labor compensation and hours from KLEMS are used to construct an aggregate wage consistent with the measure of aggregate productivity. The labor compensation variable in KLEMS includes both wage and non-wage components and reflects the compensation of the self-employed, while the hours variable is adjusted to account for self-employment. Therefore, KLEMS provides a more reliable source of aggregate compensation and aggregate hours for the economy.

Two key variables of interest are the wages of low-skill and high-skill labor. Low-skill labor includes individuals who are high school dropout, high school graduates, or have some college education. High-skill labor comprises college graduates and those with post-college degrees. To compute the composition-adjusted wage for the average high-skill and low-skill worker, the KLEMS data are merged with the distribution of demographic subgroups from the CPS.[6] Since the distribution of demographic subgroups comes from the CPS, the implied relative wage aligns with that of the CPS.

Sectors are classified into high-skill intensive sector and low-skill intensive sector based on their long-term high-skill labor income shares, as reported in Table A.1. The high-skill intensive sector includes: finance, insurance, government, health and education services, while the low-skill intensive sector includes the remaining industries. Additional details are provided in Appendix A.1.2.

Productivity is calculated as total value-added divided by total labor input in the aggregate economy or at the sector level. Labor input of low- and high-skill workers is measured in terms of efficiency hours, computed as labor compensation divided by the composition-adjusted wage of each skill group. Deflators of real productivity are KLEMS value-added prices at the aggregate and sector levels. Sectoral prices for high- and low-skill-intensive sectors are calculated as Tornqvist indexes based on the value-added shares and prices of finer industries within each sector.

2.2. Motivating facts

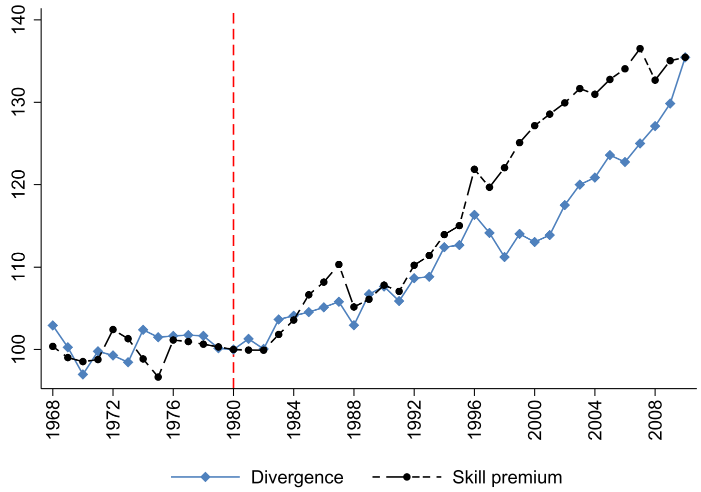
To provide a set of summary statistics regarding the stagnation in the low-skill real wage and its divergence from the aggregate labor productivity, we take the 5–year average 1978–1982 for the year 1980 and 2006–2010 for the year 2008. During this period, aggregate labor productivity increased by 60%. The low-skill real wage, on the other hand, only increased by 16% or 26% if the nominal wage is deflated by the Consumption Price Index (CPI) or the Personal Consumption Expenditure (PCE) price index, respectively. The growth in the high-skill real wage was much higher at 56% (CPI) or 70% (PCE), leading to an increase in the skill premium (ratio of high-skill to low-skill wage) from 1.44 to 1.94.

The main objective of the paper is to show that faster productivity growth in the low-skill sector has contributed to the stagnation of the low-skill real wage and its divergence from the aggregate productivity growth. The main mechanism works through both falling relative prices of the low-skill intensive sector and the reallocation of low-skill labor away from the low-skill intensive sector. This hypothesis is consistent with the data during this period. The annual labor productivity growth is 2.3% in the low-skill intensive sector and 0.1% in the high-skill intensive sector. The relative price of the high-skill intensive sector increased by 49% while the share of low-skill workers in the high-skill intensive sector increased from 14% to 21%.

It is interesting to note that the low-skill real wage was growing at about the same rate as the high-skill real wage and the aggregate labor productivity prior to 1980 before it started to lag behind both series, see Fig. 1. As shown in Fig. 2, the timing of the low-skill wage stagnation is consistent with our mechanism. Specifically, Fig. 2A shows that the rise in the relative productivity of the low-skill

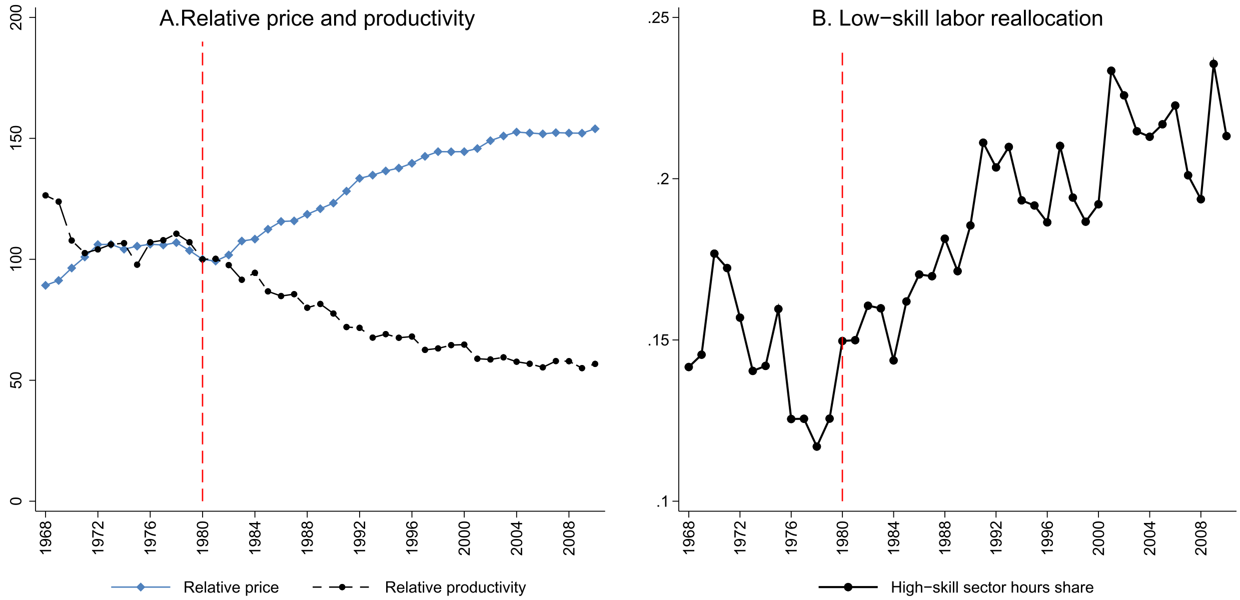
[5] More specifically, they show that the expansion of the high-skill intensive sector induces an increase in the relative wage of high-skill worker. Earlier work by Ngai and Petrongolo (2017) shows that the expansion of the intensive female sector raises the relative wage of women. Both results are related to the Stolper-Samuelson theorem.

[6] Wages are calculated as labor compensation per hour. Composition adjustment is performed using long-run hours shares across categories of age, sex, race, and education within the high-skill and low-skill labor groups. See Appendix A.1.1 for details.



Note: Divergence is the ratio of aggregate labor productivity relative to the low-skill real wage. Skill premium is the ratio of the high-skill wage relative to the low-skill wage. Low-skill is defined as education less than a university degree. Composition-adjusted wages control for age, sex, race and education within the high-skill and the low-skill. See Section 2.1 for the construction of variables. Source: World KLEMS and CPS.

Fig. 1. Low-skill real wage stagnation.



Note: Panel A shows the value-added price and real labor productivity of the high-skill sector relative to the low-skill sector, normalized to 100 in 1980. Panel B shows the share of low-skill hours in the high-skill sector. See Section 2.1 for the construction of variables and sectors. Source: World KLEMS and CPS.

Fig. 2. Relative productivity, relative prices and low-skill labor reallocation.

sector started mainly after 1980 and this is mirrored by the fall in the relative price of the low-skill sector. Fig. 2B shows that the reallocation of low-skill workers into the high-skill sector also started after 1980. These motivating figures highlight the potential importance of our multisector perspective for understanding the low-skill wage stagnation. The next section presents a model to quantify its role.

We conclude this section by highlighting the role of capital in understanding the divergence of the low-skill wage from aggregate labor productivity. The divergence can be decomposed into three factors using an accounting relationship. Starting with the definition of the labor income share $\beta y = w$, where β is the aggregate labor income share, y is the nominal aggregate labor productivity and w is the average nominal wage. Let P_Y be the aggregate output price index and P_C be the consumption price index, we can express:

$$\underbrace{\frac{y/P_Y}{w_l/P_C}}_{\text{Divergence}} = \underbrace{\left(\frac{P_C}{P_Y}\right)}_{\text{Living Cost}} \underbrace{\left(\frac{1}{\beta}\right)}_{\text{Labor Share}} \underbrace{\left(\frac{w}{w_l}\right)}_{\text{Wage Inq}} \tag{1}$$

The divergence in the low-skill real wage and aggregate productivity is attributable to three factors: (1) a rise in the relative cost of living, (2) a decline in labor share, and (3) a rise in wage inequality, measured by the ratio of the average wage relative to the low-skill wage. The relative contributions of these three factors depend on the choice of the consumption price index. If we use PCE as a measure of P_C, then the contributions of the three factors are 10%, 20% and 70%. If we use CPI instead, then the contributions are 30%, 20% and 50%.[7] The main takeaway is that all three factors are quantitatively important. The presence of capital is essential for the first two factors to exist. Without capital, both the relative price of consumption and the labor income share are equal to one.

3. The model

The economy consists of two sectors: the high-skill sector and the low-skill sector. There is a measure H of high-skill households and a measure L of low-skill households. Each household is endowed with one unit of time which is supplied to the market inelastically.

Household $i = l, h$ derives utility from consuming output from both sectors:

$$U_i = \ln c_i; \quad c_i = \left[\psi c_{il}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\psi) c_{ih}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \quad i = h, l, \tag{2}$$

where $\varepsilon < 1$ so that low-skill and high-skill goods are complements. The budget constraint is:

$$p_h c_{ih} + p_l c_{il} = w_i, \quad i = h, l, \tag{3}$$

where w_i is the wage of household i. The optimal relative consumption is derived from equating the marginal rate of substitution to the relative prices, which can be aggregated to derive the relative aggregate consumption (see Appendix A.2.1):

$$\frac{C_h}{C_l} = \left[\frac{p_l}{p_h}\left(\frac{1-\psi}{\psi}\right)\right]^{\varepsilon}, \quad C_j = Lc_{lj} + Hc_{hj}, \quad j = h, l. \tag{4}$$

The representative firm in sector $j = l, h$ uses low-skill labor, high-skill labor, and capital as inputs:

$$Y_j = A_j F_j\left(G_j\left(H_j, K_j\right), L_j\right) \tag{5}$$

$$F_j\left(G_j\left(H_j, K_j\right), L_j\right) = \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + \left(1-\xi_j\right)\left[G_j\left(H_j, K_j\right)\right]^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{6}$$

$$G_j\left(H_j, K_j\right) = \left[\kappa_j K_j^{\frac{\rho-1}{\rho}} + \left(1-\kappa_j\right) H_j^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{7}$$

where H_j and L_j are the high-skill labor and the low-skill labor used in sector j. The parameter κ_j measures the importance of capital within the capital-skill composite. The elasticity of substitution across high-skill labor and capital $\rho < 1$ captures the capital-skill complementarity.

The output of the low-skill sector can be converted into $1/\phi$ unit of capital, where ϕ is interpreted as the price of capital relative to the low-skill intensive goods.[8] The objective of the quantitative exercise is to compare the labor market changes from 1980 to 2010 instead of studying the time path. To keep the framework simple, we assume full depreciation of capital. The market clearing conditions for goods, capital, and labor are:

$$Y_l = C_l + \phi K, \quad Y_h = C_h. \tag{8}$$

$$K = K_h + K_l. \tag{9}$$

$$H_h + H_l = H; \quad L_h + L_l = L. \tag{10}$$

3.1. Firm's optimization

The optimal decision of the representative firm implies that the marginal rate of technical substitution across any two inputs is equal to the ratio of their relative prices. This implies the ratio of the high-skill labor and capital satisfies:

[7] The role of different price deflators, the declining labor income share, and difference between mean and median wages have been empirically documented as sources of the decoupling between average wage and productivity (e.g., Lawrence and Slaughter, 1993; Bivens and Mishel, 2015). Here and in the next section, we deflate output by the value-added deflator and wages by alternative consumer price deflators (See Stansbury and Summers, 2019; Greenspon et al., 2021, for a similar empirical approach).

[8] This two-sector model can be mapped into a three-sector model where the low-skill intensive sector is an aggregation of a consumption goods sector and a capital goods sector under the assumption that they have identical production functions except the sector-specific TFP index. In this environment, the relative price of capital ϕ is equal to the inverse of their relative TFPs, so a fall in ϕ is interpreted as an investment-specific technical change (Greenwood et al., 1997).

$$\frac{H_j}{K_j} = \left(\chi \delta_j\right)^{-\rho}; \qquad \delta_j \equiv \frac{\kappa_j}{1-\kappa_j}, \quad \chi \equiv \frac{w_h}{q_k}, \quad j = h, l, \tag{11}$$

where w_h is the high-skill wage and q_k is the rental price of capital. Define $\tilde{I}_j$ as the ratio of the high-skill labor income relative to the sum of high-skill labor and capital income:

$$\tilde{I}_j \equiv \frac{w_h H_j}{q_k K_j + w_h H_j} = \frac{1}{1 + \chi^{\rho-1} \delta_j{}^{\rho}}, \quad j = h, l, \tag{12}$$

where the last equality follows from the condition (11). Using the optimal condition across the high-skill and the low-skill labor, Appendix A.2.2 shows that the relative skill-intensity is:

$$\frac{H_j}{L_j} = \left(\sigma_j / q\right)^{\eta} \left(1-\kappa_j\right)^{\frac{\rho(\eta-1)}{(\rho-1)}} \tilde{I}_j^{\frac{\eta-\rho}{1-\rho}}; \quad \sigma_j \equiv \frac{1-\xi_j}{\xi_j}, \quad q \equiv \frac{w_h}{w_l}, \quad j = h, l, \tag{13}$$

where q denotes the skill premium. Define J_j as the low-skill income share and I_j as the high-skill income share in sector $j = h, l$:

$$J_j \equiv \frac{w_l L_j}{q_k K_j + w_h H_j + w_l L_j} = \left[1 + q^{1-\eta} \sigma_j^{\eta} \left[\tilde{I}_j \left(1-\kappa_j\right)^{-\rho}\right]^{\frac{\eta-1}{1-\rho}}\right]^{-1}, \tag{14}$$

$$I_j \equiv \frac{w_h H_j}{q_k K_j + w_h H_j + w_l L_j} = \left(1 - J_j\right) \tilde{I}_j. \tag{15}$$

Using (14) and (15), Appendix A.2.2 derives the sectoral labor income share as:

$$\beta_j = I_j + J_j = J_j \left(q^{1-\eta} \sigma_j^{\eta} \left[\tilde{I}_j \left(1-\kappa_j\right)^{-\rho}\right]^{\frac{\eta-\rho}{1-\rho}} + 1\right); \quad j = h, l. \tag{16}$$

3.2. Equilibrium prices and allocation

The equilibrium low-skill wage is equal to the value of the marginal product of low-skill labor MPL_{lj} in sector j, which is derived in Appendix A.2.2 as:

$$w_l = p_j MPL_{lj}; \qquad MPL_{lj} \equiv \frac{\partial Y_j}{\partial L_j} = A_j \left(J_j \xi_j^{-\eta}\right)^{\frac{1}{1-\eta}}, \tag{17}$$

Let $P_C = (\psi^{\varepsilon} p_l^{1-\varepsilon} + (1-\psi)^{\varepsilon} p_h^{1-\varepsilon})^{\frac{1}{1-\varepsilon}}$ be the aggregate consumption price index, the low-skill real wage is given as:

$$\frac{w_l}{P_c} = A_l \left(J_l \xi_l^{-\eta}\right)^{\frac{1}{1-\eta}} \frac{p_l}{P_C}; \qquad \frac{p_l}{P_C} = \left(\psi^{\varepsilon} + (1-\psi)^{\varepsilon} \left(\frac{p_l}{p_h}\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}. \tag{18}$$

The relative price is then derived from the free mobility of labor:

$$\frac{p_h}{p_l} = \left(\frac{A_l}{A_h}\right) \left(\frac{\xi_l}{\xi_h}\right)^{\frac{\eta}{\eta-1}} \left(\frac{J_h}{J_l}\right)^{\frac{1}{\eta-1}}, \tag{19}$$

which shows that faster productivity growth in the low-skill sector implies a falling relative price of the low-skill sector. This generates the negative relationship between relative price and relative productivity documented in Fig. 2.

The equilibrium conditions derived above are functions of the relative factor prices (χ, q), where q is derived as a function of χ in Appendix A.2.3:

$$q = \chi \left[\left(\frac{\phi}{A_l}\right)^{\eta-1} \xi_l^{-\eta} - \sigma_l^{\eta} \left[\left(\chi^{1-\rho} + \delta_l^{\rho}\right) \left(1-\kappa_l\right)^{\rho}\right]^{\frac{1-\eta}{1-\rho}}\right]^{\frac{1}{\eta-1}}. \tag{20}$$

Finally, Appendix A.2.3 shows that the equilibrium of the model can be summarized by solving for χ and the share of low-skill labor in the high-skill sector $(l_h \equiv L_h / L)$ using two conditions:

$$l_h = S\left(\chi; \frac{H}{L}, \frac{\phi}{A_l}\right) \equiv \frac{\frac{H}{L} q^{\eta} \sigma_l^{-\eta} \left(1-\kappa_l\right)^{\frac{\rho(\eta-1)}{1-\rho}} \tilde{I}_l^{\frac{\eta-\rho}{\rho-1}} - 1}{\left(\frac{\sigma_h}{\sigma_l}\right)^{\eta} \left(\frac{1-\kappa_l}{1-\kappa_h}\right)^{\frac{\rho(\eta-1)}{1-\rho}} \left(\frac{\tilde{I}_l}{\tilde{I}_h}\right)^{\frac{\eta-\rho}{\rho-1}} - 1}. \tag{21}$$

$$l_h = D\left(\chi; \hat{A}_{lh}, \frac{\phi}{A_l}\right) \equiv \left[1 + \frac{J_l}{J_h} \left(\frac{1}{x \beta_l} + \frac{1-\beta_h}{\beta_l}\right)\right]^{-1}, \tag{22}$$

where the relative consumption expenditure share is derived from (4) and (19):

$$x \equiv \frac{p_h C_h}{p_l C_l} = \hat{A}_{lh}^{1-\epsilon}\left(\frac{J_h}{J_l}\left(\frac{\xi_l}{\xi_h}\right)^{\eta}\right)^{\frac{1-\epsilon}{\eta-1}}; \quad \hat{A}_{lh} \equiv \frac{A_l}{A_h}\left(\frac{1-\psi}{\psi}\right)^{\frac{\epsilon}{1-\epsilon}}, \tag{23}$$

and the consumption expenditure shares are $x_l = 1/(1+x), x_h = x/(1+x)$. In a nutshell, the condition $S\left(\chi; \hat{A}_{lh}, \frac{\phi}{A_l}\right)$ is derived using the labor market clearing conditions and the firm's optimization, and the condition $D\left(\chi; \hat{A}_{lh}, \frac{\phi}{A_l}\right)$ is derived using the goods market clearing conditions and the household's optimization. These two conditions together solve for (χ, l_h) and the skill premium q is obtained from (20). Given q and χ, the low-skill wage is derived from (17) and the income shares are derived from (12), (14), and (15). Appendix A.2.3 derives the value-added shares as:

$$v_h \equiv \frac{p_j Y_j}{\sum_j p_j Y_j} = \left[1 + \left(\frac{J_h}{J_l}\right)\left(\frac{1-l_h}{l_h}\right)\right]^{-1}, \quad v_l = 1 - v_h, \tag{24}$$

which then deliver the aggregate labor income share as:

$$\beta = \beta_l v_l + \beta_h v_h. \tag{25}$$

3.3. Divergence

The accounting identity (1) shows that the divergence of the low-skill real wage from aggregate labor productivity is due to rising relative cost of living, falling labor income shares and rising wage inequality. Using the equilibrium conditions derived above, we now explain how the model can generate these three factors through faster productivity growth in the low-skill sector.

A faster productivity growth in the low-skill sector decreases the relative price of the low-skill goods (19) and increases the relative consumption share (23) given consumption complementarity ($\epsilon < 1$). This implies a reallocation of labor towards the high-skill sector (22), which acts as an endogenous skill-biased shift leading to a higher skill premium q as in Buera et al. (2022).

The relative cost of living is measured by the price of aggregate consumption relative to the price of aggregate output, P_C/P_Y. These two price indexes can be obtained by the Tornqvist method using the consumption expenditure shares x_j as weights for P_C and the value-added shares v_j as weights for P_Y. Given the consumption share of the high-skill sector exceeds its value-added share, the faster productivity growth in the low-skill sector implies a rise in the relative cost of living P_C/P_Y.[9]

The effect on the aggregate labor income share β in (25) is ambiguous for two reasons. First, it predicts a rise in the skill premium which has two opposing effects on the sectoral labor income share β_j derived in (16). More explicitly, it reduces the low-skill income share in (14) and increases the high-skill income share in (15) in both sectors. Second, there is an increase in the value-added share of the high-skill sector (v_h in (24)), which can lower the aggregate labor income share if $\beta_h < \beta_l$, and vice versa.

3.4. Low-skill wage and skill premium

The skill premium measures the high-skill wage relative to the low-skill wage. A rise in the skill premium does not necessarily imply a slower growth in the low-skill wage. In a similar vein, factors that imply a rise in the skill premium do not always imply a slower growth in the low-skill wage. Using the optimal capital-skill ratio in (11), the production function (5) can be expressed as a function of the high-skill and low-skill labor:

$$Y_j = \tilde{A}_j\left[(1-\lambda_j)H_j^{\frac{\eta-1}{\eta}} + \lambda_j L_j^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{26}$$

$$\tilde{A}_j \equiv A_j\left(\xi_j + (1-\xi_j)\left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\left(\frac{\rho}{\rho-1}\right)\left(\frac{\eta-1}{\eta}\right)}\right)^{\frac{\eta}{\eta-1}}; \lambda_j \equiv \frac{\xi_j}{\xi_j + (1-\xi_j)\left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\left(\frac{\rho}{\rho-1}\right)\left(\frac{\eta-1}{\eta}\right)}}, \tag{27}$$

which takes a similar form as the aggregate production function used in the literature (see Katz and Murphy, 1992; Heathcote et al., 2010), where a decrease in λ of the aggregate production function represents an aggregate skill-biased shift. Our model provides two *endogenous* sources for this aggregate skill-biased shift.

First, as in Buera et al. (2022), the predicted shift towards the high-skill sector implies a decrease in the aggregate λ when $\lambda_h < \lambda_l$. This between-sector skill-biased shift is shown to be an important source for the increase in the aggregate skill intensity for understanding the rise in the skill premium. Second, as in Krusell et al. (2000), falling relative price of capital implies an increase in $\tilde{I}_j$ due to capital-skill complementarity. This implies a decrease in λ_j acting as a within-sector skill-biased shift in both sectors.

Both shifts imply a rise in the skill premium but they have different effects on the low-skill wage. The between-sector shift induces a shift from the low-skill sector with high λ_l to the high-skill sector with low λ_h, so it reduces the aggregate λ contributing to a

[9] The assumption that capital is only produced by the low-skill sector helps to simplify the model but what is necessary for the consumption share of the high-skill sector to be larger than its value-added share is that the low-skill sector contributes more to the production of capital. This is supported by findings of McGrattan (2020) which confirm that our high-skill intensive sectors provide a negligible portion of tangible and intangible capital to other industries.

slower growth in the low-skill wage. The within-sector shift, through rising $\tilde{I}_j$, reduces λ_j in both sectors but this effect is offset by the implied rise in the effective productivity $\tilde{A}_j$ due to the capital-skill complementarity (i.e. $\rho < 1$, see (27)). Thus the falling relative price of capital contributes to a rise in the skill premium but not necessarily to the low-skill wage stagnation.

There are other sources of within-sector skill-biased shifts that can lower λ_j through falling production weights κ_j and ξ_j. A fall in κ_j can reflect a skill-biased organizational change that increases the importance of human capital (Caroli and Van Reenen, 2001).[10] Similar to the role of the falling relative price of capital, a fall in κ_j reduces λ_j but also implies a rise in the effective productivity $\tilde{A}_j$, resulting in an ambiguous effect on the low-skill wage. A fall in ξ_j, however, implies a fall in both λ_j and $\tilde{A}_j$ when high-skill and low-skill labor are good substitutes ($\eta > 1$). Thus, it can contribute to both a rise in the skill premium and the low-skill wage stagnation. The decline in the production weights for low-skill workers can be due to the displacement effect from automation in the task-based model of Acemoglu and Restrepo (2018) and the outsourcing of tasks performed by low-skilled workers in Grossman and Rossi-Hansberg (2008).

The skill-biased shifts discussed above can be put into the three classes of technical changes discussed in Johnson (1997). The fall in κ_j is an *intensive skill-biased technical change* that raises the marginal product of high-skill labor without directly affecting the marginal product of low-skill labor; thus it contributes to the skill premium but has little effect on the growth of the low-skill wage. The fall in ξ_j is an *extensive skill-biased technical change* that increases the marginal product of high-skill labor and lowers the marginal product of low-skill labor, thus contributing to both the increase in skill premium and the stagnation of low-skill wage. What is interesting is that the increase in A_h and A_l, which are *skills-neutral technical changes* at the sectoral level, becomes skill-biased at the aggregate level due to different factors intensities across sectors, affecting both the skill premium and the low-skill wage growth.

3.5. Demand shift towards high-skill intensive goods

In addition to uneven productivity growth, a demand shift towards high-skill intensive goods can also act as a source for the between-sector skill-biased shift. This demand shift can be induced by rising income if high-skill intensive goods have a higher income elasticity. As shown by Comin et al. (2021), a fall in the preference parameter ψ in the homothetic CES utility function (2) can capture this income effect in a more general non-homothetic CES utility function.[11] Thus, by examining the effect of a fall in ψ, we can learn about the effect of a demand shift towards the high-skill sector on the low-skill wage.

Using (23), a fall in ψ implies an increase in $\hat{A}_{lh}$ and a rise in relative expenditure; thus, it has a similar effect on the skill premium as the increase in the relative productivity A_l/A_h. However, it does not have a direct effect on the relative prices of the high-skill intensive sector as shown in equation (19), nor the low-skill real wage in (18).[12] Its contribution to the divergence is through the increase in the skill premium, which is similar to the effect of a skill-biased shift through ξ_j. Thus, we let the calibration of ξ_j pick up its role as a skill-biased demand shift.

3.6. Role of changing relative prices

Before turning to the quantitative results, it is worth noting that falling relative prices of the low-skill goods can contribute to the low-skill wage stagnation even in the absence of the reallocation of low-skill labor. This can be seen in a special case where sector l only uses low-skill labor and sector h only uses high-skill labor, i.e. $\xi_l \to 1, \xi_h \to 0, \kappa_h \to 0$. There is no labor reallocation in this special case, allowing us to focus solely on the role of changing relative prices. The equilibrium outputs are $Y_l = A_l L$ and $Y_h = A_h H$ and wages are $w_l = p_l A_l$ and $w_h = p_h A_h$. The low-skill real wage is simply $w_l = A_l p_l / P_C$ as a special case of (18), where the relative price is derived from substituting the goods market clearing condition $C_j = Y_j$ into (4):

$$\frac{p_l}{p_h} = \frac{\psi}{1-\psi}\left(\frac{A_h H}{A_l L}\right)^{1/\varepsilon}. \tag{28}$$

An increase in the productivity of the low-skill sector A_l raises the marginal product of the low-skill labor (also equal to A_l), which has a direct positive effect on the real wage. However, the total effect depends on the relative price, which is the crucial difference from a one-sector model where the low-skill real wage simply equals the marginal product of labor. As shown in (28), the relative price of the low-skill sector depends negatively on its relative productivity, as long as the two goods are not perfect substitutes. If productivity growth is the same across sectors, there will be no change in relative price. In this case, the increase in the real wage for low-skills will be the same as the direct effect of the higher A_l, which will be the case in a one-sector model. However, if productivity growth is faster in the low-skill sector (A_l/A_h increases), the decrease in the relative price of the low-skill sector dampens the positive effect of productivity on the real low-skill wage. In other words, although low-skill workers produce more output, the increase in their physical productivity is offset by the decrease in the price of the goods they produce relative to their consumption basket.[13]

[10] In general, it contributes to the skill-enhancing changes in the standard canonical skill-biased technical change model (Katz and Murphy, 1992) without capital.

[11] This can be seen explicitly from comparing the relative expenditure derived in (4) with the relative expenditure derived from a non-homothetic CES utility function in Comin et al. (2021).

[12] It has an equilibrium effect on the relative price through the rise in q by changing J_h/J_l in (19), but the effect is small as it depends on the difference between the parameters ξ_h and ξ_l as shown in (14).

[13] In other words, specializing in sectors with faster productivity growth works against low-skill workers, as the output they produce is getting cheaper over time. This has a similar flavor, but the mechanism is different from the early trade literature on immiserizing growth, where faster productivity growth results in a country being worse off due to deteriorating terms of trade (Bhagwati, 1958).

Table 1
Data targets.

Level		1980	2008
Low-Skill Worker Income Share			
Total Economy	J	0.41	0.28
High-Skill Sector	J_h	0.23	0.21
Low-Skill Sector	J_l	0.46	0.32
High-Skill Worker Income Share			
Total Economy	I	0.17	0.28
High-Skill Sector	I_h	0.33	0.44
Low-Skill Sector	I_l	0.12	0.21
Skill Premium	q	1.44	1.94
Growth (% p.a.)			
Aggregate Real Labor Productivity	y/P_Y	-	1.7
Price (Relative to Low-Skill Sector)			
High-Skill Sector	p_h/p_l	-	1.4
Capital	ϕ	-	-0.5

Note: High-skill are those with college or a higher degree. Skill premium is the ratio of high-skill wage relative to low-skill wage.

On the other hand, increasing productivity in the high-skill sector can boost the low-skill real wage by increasing the relative price of low-skill goods. Therefore, an important message from the multi-sector perspective is that the source of aggregate productivity growth is important for understanding low-skill wage stagnation and its divergence from aggregate productivity.

4. Quantitative results

The model is calibrated to match the key features of the US economy. To evaluate the quantitative role of uneven productivity growth, the baseline also includes changes in the relative price of capital, the production weights of low-skill labor and high-skill labor, and the relative supply of high-skill labor. The productivity parameters are calibrated to match the increase in the relative price of the high-skill intensive sector and the aggregate labor productivity growth. The production weights are set to match the sectoral income shares, while the relative supply of high-skill labor is set to match the aggregate income of the high-skill labor relative to the low-skill labor. The predictions in the baseline are driven by changes in five sets of parameters: $\hat{A}_{lh}$ in equation (23), the relative price of capital ϕ, the production weights $\left\{\xi_l, \xi_h, \kappa_l, \kappa_h\right\}$ in (5), and the relative supply of the high-skill labor H/L.[14]

4.1. Data targets

The construction of the data targets reported in Table 1 is summarized in Section 2 and described in detail in Appendix A.1. Data from the 5–year average 1978–1982 were used for the year 1980 and 2006–2010 for the year 2008. During this period, the high-skill income share (I_j) increases while the low-skill income share (J_j) decreases in both sectors. The total labor income share ($\beta_j = I_j + J_j$) falls in the low-skill sector but increases in the high-skill sector, and the aggregate labor income share ($I + J$) falls.

The annual growth rate of the aggregate real labor productivity is 1.7% and the relative price of the high-skill sector is 1.4%. Using the ratio of P_K/P_Y from the BEA and the ratio P_Y/p_l from the KLEMS, the price of capital relative to the low-skill sector (ϕ) declines at 0.5% per year.[15]

4.2. Calibration

The elasticity of substitution across high-skill and low-skill labor, $\eta = 1.4$, is taken from Katz and Murphy (1992). The elasticity of substitution across capital and high-skill labor, $\rho = 0.67$, is taken from Krusell et al. (2000). There is no direct estimate of the elasticity of substitution across high-skill and low-skill goods, ε. The literature on structural transformation finds that the elasticity of substitution across agriculture, manufacturing, and services is close to zero (Herrendorf et al., 2013). Given that we re-group these three sectors into two sectors, this likely implies a higher degree of substitution. Ngai and Pissarides (2008) report a range of estimates for the price elasticity of services from -0.3 to 0, which is informative but not an exact estimate for $-\varepsilon$, the price elasticity of the high-skill sector in our model. Based on these estimates, we use $\varepsilon = 0.2$ as our baseline value for the elasticity of substitution across the two sectors.[16]

[14] Given the definition of $\hat{A}_{lh}$ in equation (A.2), we do not need to separate the preference parameter ψ from A_l/A_h to solve for the model.

[15] The price of capital is calculated as the investment in total fixed assets divided by the chain-type quantity index for investment in total fixed assets (Tables 1.5 and 1.6 of the BEA's Fixed Assets Accounts). It is worth noting that the growth of P_Y in KLEMS is 2.94%, which is almost identical to that of BEA at 2.86%.

[16] The quantitative results are not sensitive to small changes in the values of elasticity parameters (ε, η, ρ).

Table 2
Calibrated parameters.

A. Parameters from the literature				
Parameters			Values	Source
ε			0.2	Benchmark value, see main text
ρ			0.67	Krusell et al. (2000)
η			1.4	Katz and Murphy (1992)
B. Calibrated parameters				
Parameters	1980	2008	Growth (% p.a.)	Target
ϕ			-0.50	Price of capital relative to the low-skill goods
A_l			1.09	Aggregate real labor productivity
A_{lh}			1.82	Relative price of the high-skill sector
ξ_l	0.33	0.25	-0.93	Sectoral income share. See Appendix A.2.4
ξ_h	0.20	0.19	-0.13	Sectoral income share. See Appendix A.2.4
κ_l	0.74	0.69	-0.22	Sectoral income share. See Appendix A.2.4
κ_h	0.41	0.33	-0.79	Sectoral income share. See Appendix A.2.4
H/L	0.29	0.50	1.92	Relative aggregate labor income shares I_t/J_t

The relative supply of high-skill labor (H/L) is obtained from the data on the skill premium and income shares (q_t, I_t, J_t).[17] Appendix A.2.4 reports the calibration procedure for the remaining parameters. The calibration strategy is as follows: production weights (ξ_j, κ_j) are set to match sectoral income shares in the data for any given value of ϕ/A_l. To simplify the explanation, denote 1980 as period 0 and 2008 as period T. We show that ϕ_0/A_{l0} can be normalized to 1 and obtain all production weights in period 0. Using these parameters, condition (21) implies a value of l_{h0}, and condition (22) implies a value of $\hat{A}_{lh0}$ given q_0. For a given level of A_{lT}/A_{l0}, data on the decrease in ϕ_t implies a value for ϕ_T/A_{lT}, which assigns all production weights in period T. We then set the change in A_{lhT}/A_{lh0} to match the increase in the relative price of the high-skill sector. Finally, A_{lT}/A_{l0} is adjusted to match the change in aggregate labor productivity deflated by the price of the low-skill sector.

Table 2 reports the calibrated parameters. The implied annual growth of ϕ, A_{lh}, A_l, H/L, and production weights (κ_j, ξ_j) are reported in Panel B of Table 2.[18] Matching the rise in the relative price of the high-skill sector implies faster productivity growth in the low-skill sector.[19] Matching the relative aggregate income shares of high-skill and low-skill labor implies a rise in the relative supply of high-skill labor. Matching the sectoral income shares, on the other hand, requires changes in the production weights reflecting other sources of skill-biased shifts. The growth in relative productivity A_l/A_h is governed by the increase in the relative price of the high-skill sector, which is equal to 49% as reported in Section 2.[20] This, together with the observed growth in aggregate productivity, determines the growth in the sectoral productivity parameters (A_l, A_h). It is reassuring to report that the baseline calibration implies labor productivity growth of 2.2% for the low-skill sector and -0.2% for the high-skill sector, closely matching the observed 2.3% and 0.1% in the data.

4.3. *Results on sectoral shares and skill premium*

As reported in row 2 of Table 3, the baseline does a good job of matching the increase in the skill premium, the pattern of sectoral reallocation, and the changes in labor share in each sector. The remaining rows of Table 3 examine each of the five forces that drive these changes by shutting them down one at a time: the uneven sectoral productivity growth (higher A_l/A_h) in row 3, the falling relative price of capital (ϕ) in row 4, the falling production weights of low-skill labor (ξ_l, ξ_h) in row 5, the rising production weights of high-skill labor within the capital-skill composite (κ_l, κ_h) in row 6, and the increase in the relative supply of high-skill labor (higher H/L) in row 7. It is important to note that in order to match the increase in aggregate productivity at 60%, the growth in A_l has to be adjusted in each of row 3 to 7. More specifically, for row 3, fixing the relative productivity A_l/A_h at the 1980 level requires the same productivity growth in both sectors. This implies a large growth in A_h if we keep the growth in A_l as in the baseline, which would imply a larger increase in aggregate productivity (85% instead of 60%). Thus, we lower the growth in A_l so that the implied change in aggregate productivity growth is the same as in the baseline at 60%.

The results confirm the intuition that uneven productivity growth (row 3) is crucial for sectoral reallocation. In a world with balanced productivity growth, there would be no reallocation of low-skill labor, and the value-added shares of the high-skill sector would have fallen. While the fall in the production weights of low-skill labor (row 5) is essential for the decrease in the labor share

[17] The H_j and L_j are not the raw market hours by the high-skill and low-skill workers in the data. The composition-adjusted high-skill hours H_j in sector j are computed as the high-skill income in sector j divided by the composition-adjusted high-skill wage; similarly for L_j.

[18] The implied negative growth in κ_j does not necessarily indicate a decrease in the usage of capital. It only implies a fall in the input weight of capital in the capital-skill composite.

[19] The calibration implies that A_h is falling, which can be understood using the findings of Aum et al. (2018) and Bárány and Siegel (2021). The former paper finds negative productivity growth for high-skill occupations (professional and management), while the latter finds negative growth for abstract occupations. Their findings could be the source of the drop A_h given that these occupations are concentrated in the highly skilled intensive sector.

[20] If we were to halve the increase in the relative price of the high-skill sector, the uneven productivity growth across sectors would remain quantitatively important, albeit to a smaller extent. This result is available upon request.

Table 3
Sectoral shares and the skill premium.

			Sectoral reallocation			Sectoral labor share		Skill premium
			l_h	h_h	v_h	β_l	β_h	q
	Data 1980		0.14	0.46	0.24	0.59	0.56	1.44
(1)	Data 2008		0.21	0.46	0.29	0.53	0.65	1.94
(2)	Model 2008		0.20	0.45	0.28	0.53	0.65	1.92
Counterfactual (fixing each parameter to its 1980 value)								
(3)	*Relative productivity*	A_l/A_h	0.15	0.36	0.21	0.52	0.64	1.79
(4)	*Relative capital price*	ϕ	0.19	0.44	0.27	0.52	0.62	1.71
(5)	*Low-skill weights*	ξ_l, ξ_h	0.18	0.52	0.31	0.59	0.64	1.51
(6)	*Capital weights*	κ_l, κ_h	0.19	0.42	0.26	0.49	0.59	1.68
(7)	*Relative skill supply*	H/L	0.24	0.48	0.31	0.56	0.68	3.19

Note: The productivity growth of the low-skill sector is adjusted in row 3 to row 7 to match the 60% increase in the aggregate productivity.

Table 4
Divergence: low-skill real wage and aggregate real labor productivity (percentage change, 1980–2008).

				Factors of divergence		
			Divergence $\frac{y/P_Y}{w_l/P_C}$	Wage inequality w/w_l	Labor share β	Living cost P_C/P_Y
(1)	Data		27 (38)	19	-3.4	2.8 (12)
(2)	Model		34	19	-3.8	8.2
Counterfactual (fixing each parameter to its 1980 value)						
(3)	*Relative productivity*	A_l/A_h	19	15	-5.9	-2.7
(4)	*Relative capital price*	ϕ	29	12	-6.0	7.8
(5)	*Low-skill weights*	ξ_l, ξ_h	10	6.2	4.9	8.7
(6)	*Capital weights*	κ_l, κ_h	37	12	-10	9.5
(7)	*Relative skill supply*	H/L	47	36	2.4	11

Note: Divergence is measured as the percentage change in the ratio of real labor productivity divided by the low-skill real wage. The three factors of the divergence are shown in (1). For the data row, the real wage is calculated using PCE as P_C and the number in bracket uses CPI. The productivity growth of the low-skill sector is adjusted in row 3 to row 7 to match the 60% increase in the aggregate productivity.

in the low-skill sector, the rise in the production weights of high-skill labor (row 6) is important for the increase in the labor share in the high-skill sector. Not surprisingly, the increase in the relative supply of high-skill labor contributes to lowering the skill premium.

Consistent with the previous literature, all mechanisms are important for the increase in the skill premium: uneven productivity growth (Buera et al., 2022), a falling relative price of capital (Krusell et al., 2000), the falling production weights of low-skill workers (Goldin and Katz, 2009), and the increasing production weights of high-skill labor (Katz and Murphy, 1992). However, as discussed in Section 3.4, these mechanisms can have different implications on wage-productivity divergence and the growth of low-skill wages.

4.4. Results on divergence

Table 4 presents the results on the wage-productivity divergence and the three contributing factors shown in (1): wage inequality, aggregate labor share, and relative cost of living. Since the KLEMS data do not contain information on consumption, we take P_C/P_Y as the ratio of implicit deflators of PCE and GDP from the BEA. This implies that P_C/P_Y increased by 2.8%. If we were to use the CPI, the increase in P_C/P_Y would be 11.5%. This alternative value would imply a larger divergence and slower real wage growth in the data, but does not affect other rows. Due to the concern that CPI tends to bias the increase in the cost of living (Boskin et al., 1998), we use the P_C/P_Y implied by the PCE deflator as the main data moment for comparison but keep those implied by the CPI in brackets.

Row 1 of Table 4 reports an empirical decomposition for the accounting identity in equation (1). During this 30-year period, the negative forces imposed by the rising relative cost of living, growing wage inequality, and falling aggregate labor income share largely offset the impact of rising productivity on low-skill real wages. The rise in the relative cost of living contributes 10% ($=2.8/27$) of the divergence, the increase in wage inequality contributes 70% ($=19/27$), and the fall in the aggregate labor income share accounts for the remaining 20%. If the CPI is used, the contribution of the relative cost of living increases to 30% while the contribution of the rise in wage inequality reduces to 50%.

The baseline (row 2) can account for all the rise in wage inequality and the fall in the aggregate labor share, given it matches the skill premium, sectoral shares, and sectoral labor shares in Table 3. It over-predicts (under-predicts) the relative cost of living, thus slightly over-predicts (under-predicts) the divergence, if PCE (CPI) is used as the consumption deflator.

Table 5
Low-skill real wage (percentage change, 1980–2008).

			Low-skill real wage	MPL_l		Relative price
			w_l/P_C	w_l/p_l	w_l/p_h	p_h/p_l
(1)	Data		26 (16)	44	-3.4	49
(2)	Model		20	44	-3.4	matched
Counterfactual (fixing each parameter to its 1980 value)						
(3)	***Relative productivity***	A_l/A_h	**35**	**27**	**48**	**-15**
(4)	*Relative capital price*	ϕ	24	47	1.4	45
(5)	***Low-skill weights***	ξ_l, ξ_h	**45**	**79**	**15**	**56**
(6)	*Capital weights*	κ_l, κ_h	17	43	-7.0	54
(7)	*Relative skill supply*	H/L	8.7	40	-18	70

Note: For the data row, the low-skill real wage is calculated using PCE as P_C and the number in bracket is when CPI is used as P_C. MPL_l is the marginal product of low-skill labor. The productivity growth of the low-skill sector is adjusted in row 3 to row 7 to match the 60% increase in the aggregate productivity.

Row 3 and row 5 demonstrate that the faster productivity growth of the low-skill sector and the falling production weights of low-skill labor (especially the fall in the low-skill sector) are the two most important factors for the divergence. In their absence, the predicted divergence would be reduced to almost half and a third of the baseline, respectively. However, the two mechanisms work through different channels. Although both contribute to predicting higher wage inequality, uneven productivity is important for the increase in the relative cost of living, while the fall in low-skill production weights is important for the fall in the aggregate labor share. The result that sectoral reallocation induced by uneven productivity does not contribute to the fall in aggregate labor share is consistent with the finding that the fall in the aggregate labor share in the U.S. is primarily a within-industry phenomenon (Karabarbounis and Neiman, 2014; Elsby et al., 2013; Hubmer, 2023).

The increase in the relative supply of high-skill labor in row 7 plays an important role in wage inequality. In its absence, the increase in wage inequality would have doubled, but the labor share would have increased.[21] The latter offsets some of the increase in divergence implied by the higher wage inequality. The falling relative price of capital in row 2 also contributes to the divergence by predicting a rise in wage inequality. Finally, the increasing weight of high-skill labor through falling κ (row 6) has an insignificant impact on the divergence. In its absence, wage inequality would have increased by less, while the labor share would have fallen by more, generating two opposing effects on the divergence.

4.5. Results on the low-skill wage stagnation

While Table 4 shows that all parameters (except κ_j) are important for the divergence, Table 5 reveals that only two factors are responsible for low-skill wage stagnation: the faster productivity growth of the low-skill sector (row 3) and the falling production weights of low-skill labor (row 5). In the absence of these two factors, the percentage increase in the low-skill real wage would have been more than double.

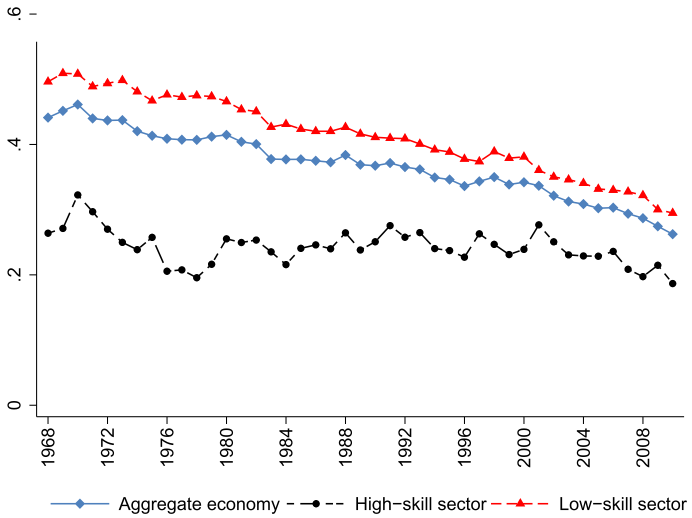
The key difference between row 3 and row 5 is their different implications for the marginal product of low-skill labor, $MPL_{lj} = w_{lj}/p_j$. In the data, MPL_l increased by 44% in the low-skill sector but decreased in the high-skill sector (due to the increase in the relative price of the high-skill sector). Uneven productivity growth is the main mechanism to deliver this result. In its absence, MPL_l would have increased more in the high-skill sector. Another difference between the two mechanisms is the predicted timing when the low-skill wage lags behind the high-skill wage and aggregate productivity (see Fig. 1). As discussed in Section 2, the uneven productivity growth mechanism is consistent with the beginning of the increase in the skill premium and the divergence that started in 1980. On the other hand, the production weights of low-skill labor (ξ_l, ξ_h) are determined by the low-skill income shares (J_l, J_h), which have been falling throughout 1968–2010 (see Fig. 3).

Finally, the increase in the relative supply of high-skilled labor (row 7) increases the growth of the low-skill real wage by increasing MPL_l in both sectors. In its absence, the growth in the low-skill real wage would have been halved. Consistent with the two opposing effects discussed in Section 3.4, the falling relative price of capital and the growth in the production weight of high-skill labor have not had a significant impact on the low-skill real wage, despite their important role in predicting the rise in the skill premium. These quantitative exercises demonstrate that factors important for the increase in the skill premium do not necessarily contribute to low-skill wage stagnation.

4.6. Sources of aggregate productivity growth

As discussed in Section 3.6, an important message from the multisector perspective is that the source of aggregate productivity growth is crucial to understanding low-skill wage stagnation. This can be seen by comparing row 3 with row 2 in Tables 4 and

[21] Its impact on the labor share is due to capital-skill complementarity, where a higher relative supply of high-skill labor increases the capital income share.



Note: The figure shows the share of low-skill labor income in the value-added of aggregate economy, the high-skill sector, and the low-skill sector. See Section 2.1. Source: World KLEMS and CPS.

Fig. 3. Trends in low-skill labor income share.

5. In the baseline (row 2), the aggregate productivity is driven purely by the growth of productivity in the low-skill sector.[22] The counterfactual exercise in row 3 instead assumes that the increase in aggregate productivity growth is due to *balanced* productivity growth in both sectors. This removes the rise in the relative price of the high-skill sector, predicting a much lower divergence in Table 4 because it predicts a much higher growth in the low-skill real wage in Table 5. This suggests that the lack of productivity growth in the high-skill sector played an important role in the observed low-skill wage stagnation and the future of the low-skill wage relies on improving the productivity growth of the high-skill sector.

5. Conclusion

Despite predominantly working in sectors with fast productivity growth, low-skill workers experience slow real wage growth that lags behind aggregate labor productivity. We argue that this phenomenon is attributable to the declining relative price of low-skill sectors, driven by their faster productivity growth.

A key insight from our multisector perspective is the importance of the source of aggregate labor productivity growth. When it originates in low-skill intensive sectors, it contributes to low-skill real wage stagnation and its divergence from aggregate labor productivity. Conversely, when it comes from high-skill intensive sectors or results from a balanced increase across both sectors, it can simultaneously boost the growth of low-skill real wages and aggregate labor productivity. In light of recent developments in artificial intelligence, which are expected to enhance productivity growth in high-skill intensive services, our view is that such development can accelerate low-skill real wage growth by decelerating the increase in the relative price of high-skill services.

Appendix A

A.1. Data appendix

A.1.1. Wages

We use March Current Population Survey Annual Social and Economic Supplement (ASEC) data from 1978 to 2012. Our sample includes wage and salary workers with a job aged 16-64, who are not students, retired, or in the military. The wage variable is based on annual wage income and hours variable is based on annual hours worked which is the product of weeks worked in the year preceding the survey and hours worked in the week prior to the survey. Top-coded components of annual wage income are multiplied by 1.5. Workers with weekly wages below $67 in 1982 dollars are dropped.

To obtain aggregate wage that is consistent with the measure of aggregate productivity, we use the aggregate labor compensation and aggregate hours from KLEMS. More specifically, to compute the composition-adjusted wage for the average high-skill worker and the average low-skill worker, we merge KLEMS 2013 data on total labor compensation and hours with the distribution of demographic subgroups in the CPS. We form 120 subgroups based on two sexes, two race groups (white and non-white), five education levels and

[22] As explained in the calibration Section 4.2, the lack of productivity growth in the high-skill sector was the result of matching the increase in the relative price of the high-skill sector and the growth of aggregate labor productivity growth simultaneously.

Table A.1
High-skill income shares by industry, 1980–2010 average.

Industry	Code	High-skill share in Value-added	High-skill share in Labor income
Agriculture, Hunting, Forestry and Fishing	AtB	10	19
Mining and Quarrying	C	11	32
Total Manufacturing	D	20	31
Electricity, Gas and Water Supply	E	9	30
Construction	F	14	16
Wholesale and Retail Trade	G	22	30
Hotels and Restaurants	H	14	18
Transport and Storage and Communication	I	16	25
Financial Intermediation	**J**	**33**	**55**
Real Estate, Renting and Business Activity	K	21	55
Public Admin	**L**	**29**	**40**
Education	**M**	**58**	**77**
Health and Social Work	**N**	**39**	**49**
Other Community, Social and Personal Services	O	23	31
Private Households with Employed Persons	P	16	16
All Industries	TOT	25	40

Note: The table reports the share of high-skill workers in total value-added and total labor income by industry. High-skill is defined as education greater than or equal to college degree. Labor income reflects total labor costs which includes compensation of employees, compensation of self-employed, and taxes on labor. Source: April 2013 Release of the World KLEMS for the U.S.

six age categories (16-24, 25-29, 30-39, 40-49, 50-59, 60-64 years). Low-skill includes high school dropout, high school graduate, and some college; high-skill includes college graduates and post-college degree categories. Compensation for each subgroup is calculated as compensation share (from CPS) times total compensation (from KLEMS). The hours worked of each subgroup is calculated in a similar way. The wage for each subgroup is then calculated as total compensation divided by total hours. The aggregate low-skill and high-skill wages are calculated as the average of the relevant subgroups using their long-run (1980–2010) hours shares as weights. It is important to note that the labor compensation variable of KLEMS includes both wage and non-wage components (supplements to wages and salaries) of labor input costs as well as reflecting the compensation of the self-employed, and hours in KLEMS are adjusted for the self-employed. Thus, KLEMS provides a more reliable source of aggregate compensation and aggregate hours in the economy. This procedure is equivalent to rescaling the CPS total hours and total wage income to sum up to KLEMS total.

A.1.2. Industry data and mapping

The March 2017 Release of the World KLEMS database reports industry value-added, price indexes, labor compensation, and capital compensation. The data are reported using the North American Industry Classification System (NAICS), which is the standard used by Federal statistical agencies in classifying business establishments in the U.S.

To classify sectors into the high-skill intensive sector and the low-skill intensive sector, we use the April 2013 Release of the World KLEMS, which provides a labor input file that allows the computation of the low-skill and high-skill workers' shares in labor compensation and value-added. High-skill is defined as education greater than or equal to a college degree. Table A.1 reports the long-run (1980–2010) average of the share of high-skill labor in the total value-added and total labor income for 15 one-digit industries. A sector is included in the high-skill intensive sector if the long-run high-skill labor income share out of the total labor income and the total value-added are above the total economy average. The high-skill intensive sector includes finance, insurance, government, health, and education services (codes J, L, M, N), and the remaining industries are grouped into the low-skill intensive sector. Our mapping across KLEMS 2013 and KLEMS 2017 is provided in Table A.2. Using this classification we map the 65 NAICS industries of the KLEMS 2017 Release into the two broad sectors for our quantitative analysis.

A.2. Model appendix

A.2.1. Household optimization

Equating the marginal rate of substitution to the relative price:

$$\frac{c_{ih}}{c_{il}} = \left[\frac{p_l}{p_h}\left(\frac{1-\psi}{\psi}\right)\right]^{\varepsilon}, \tag{A.1}$$

thus the relative consumption share is given by

$$x \equiv \frac{p_h c_{ih}}{p_l c_{il}} = \left(\frac{p_h}{p_l}\right)^{1-\varepsilon}\left(\frac{1-\psi}{\psi}\right)^{\varepsilon}. \tag{A.2}$$

Using the budget constraint to derive individual's demand:

$$p_l c_{il} = x_l w_i; \quad p_h c_{ih} = x_h w_i; \quad x_l \equiv \frac{1}{1+x}, x_h \equiv \frac{x}{1+x}, \tag{A.3}$$

Table A.2
Industry mapping.

NACE (KLEMS 2013)	NAICS (KLEMS 2017)
AtB & C	Farms, Forestry, Fishing, and Related Activities, Oil and Gas Extraction, Mining, Except Oil and Gas, Support Activities for Mining
D	Wood Products, Nonmetallic Mineral Products, Primary Metals, Fabricated Metal Products, Machinery, Computer and Electronic Products, Electrical Equipment, Appliances, and Components, Motor Vehicles, Bodies and Trailers, and Parts, Other Transportation Equipment, Furniture and Related Products, Miscellaneous Manufacturing, Food and Beverage and Tobacco Products, Textile Mills and Textile Product Mills, Apparel and Leather and Allied Products, Paper Products, Printing and Related Support Activities, Petroleum and Coal Products, Chemical Products, Plastics and Rubber Products
E	Utilities
F	Construction
G	Wholesale Trade, Retail Trade
H	Accommodation, Food Services and Drinking Places
I	Air Transportation, Rail Transportation, Water Transportation, Truck Transportation, Transit and Ground Passenger Transportation, Pipeline Transportation, Other Transportation and Support Activities, Warehousing and Storage, Publishing Industries, Except Internet (Includes Software), Motion Picture and Sound Recording Industries, Broadcasting and Telecommunications, Data Processing, Internet Publishing, and Other Information Services
J	Federal Reserve Banks, Credit Intermediation, and Related Activities, Securities, Commodity Contracts, and Investments, Insurance Carriers and Related Activities, Funds, Trusts, and Other Financial Vehicles
K	Real Estate, Rental and Leasing Services and Lessors of Intangible Assets, Legal Services, Computer Systems Design and Related Services, Miscellaneous Professional, Scientific, and Technical Services, Management of Companies and Enterprises, Administrative and Support Services, Waste Management and Remediation Services
L & M & N	Educational Services, Ambulatory Health Care Services, Hospitals and Nursing and Residential Care Facilities, Social Assistance, Federal General Government, Federal Government Enterprises, State and Local General Government, State and Local Government Enterprises
O & P	Performing Arts, Spectator Sports, Museums, and Related Activities, Amusements, Gambling, and Recreation Industries, Other Services, Except Government

Note: The table shows the mapping of KLEMS 2013 industries to KLEMS 2017. The description of KLEMS 2013 industries is provided in Table A.1.

Aggregating across households to obtain (4).

A.2.2. Equilibrium prices

Equating the marginal rate of technical substitution to the relative wage:

$$q=\sigma_j\left(1-\kappa_j\right)\left(\frac{L_j}{H_j}\right)^{\frac{1}{\eta}}\left(\frac{G_j\left(H_j,K_j\right)}{H_j}\right)^{\frac{\eta-\rho}{\rho\eta}};\qquad \sigma_j\equiv\frac{1-\xi_j}{\xi_j} \tag{A.4}$$

where, using equation (11):

$$\frac{G_j\left(H_j,K_j\right)}{H_j}=\left[\kappa_j\left(\frac{K_j}{H_j}\right)^{\frac{\rho-1}{\rho}}+\left(1-\kappa_j\right)\right]^{\frac{\rho}{\rho-1}}=\left(1-\kappa_j\right)^{\frac{\rho}{\rho-1}}\left(\delta_j^{\rho}\chi^{\rho-1}+1\right)^{\frac{\rho}{\rho-1}}=\left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\frac{\rho}{\rho-1}}. \tag{A.5}$$

Substituting into (A.4) to obtain (13). Given $I_j=\left(1-J_j\right)\tilde{I}_j$, using (12) and (14),

$$I_j=\frac{\tilde{I}_j}{1+q^{\eta-1}\sigma_l^{-\eta}\left[\tilde{I}_j\left(1-\kappa_j\right)^{-\rho}\right]^{\frac{\eta-1}{\rho-1}}} \tag{A.6}$$

Using (12) and (14), β_j in (16) is obtained from $\beta_j=I_j+J_j=\left(1-J_j\right)\tilde{I}_j+J_j$.

Equilibrium low-skill wage w_l: Using the production function:

$$\frac{F_j\left(G\left(H_j,K_j\right),L_j\right)}{L_j}=\left(\left(1-\xi_j\right)\left(\frac{G_j}{L_j}\right)^{\frac{\eta-1}{\eta}}+\xi_j\right)^{\frac{\eta}{\eta-1}}=\xi_j^{\frac{\eta}{\eta-1}}\left(\sigma_j\left(\frac{G_j}{H_j}\right)^{\frac{\eta-1}{\eta}}\left(\frac{H_j}{L_j}\right)^{\frac{\eta-1}{\eta}}+1\right)^{\frac{\eta}{\eta-1}}.$$

Substituting (A.5) and (13) to obtain:

$$\frac{F_j}{L_j}=\xi_j^{\frac{\eta}{\eta-1}}\left(\sigma_j\left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\frac{\rho}{\rho-1}\left(\frac{\eta-1}{\eta}\right)}\left(q^{-\eta}\sigma_j^{\eta}\left(1-\kappa_j\right)^{\frac{\rho(\eta-1)}{(\rho-1)}}\tilde{I}_j^{\frac{\eta-\rho}{1-\rho}}\right)^{\frac{\eta-1}{\eta}}+1\right)^{\frac{\eta}{\eta-1}}=\left(\frac{\xi_j}{J_j}\right)^{\frac{1}{\eta-1}}, \tag{A.7}$$

The low-skill real wage (17) is obtained from knowing $\partial F_j/\partial L_j=A_j\xi_j(F_j/L_j)^{1/\eta}$.

A.2.3. Sectoral allocation

Using the definition $\chi = w_h/q_k, q = w_h/w_l$, and $\phi = q_k/p_l$, equation (17) can be rewritten as

$$\chi = \frac{qA_l}{\phi}\left(J_l \xi_l^{-\eta}\right)^{\frac{1}{1-\eta}}. \tag{A.8}$$

Using (14) to derive:

$$\chi = q\xi_l^{\frac{\eta}{\eta-1}}\frac{A_l}{\phi}\left[1+q^{1-\eta}\sigma_l^{\eta}\left[\tilde{I}_l\left(1-\kappa_l\right)^{-\rho}\right]^{\frac{\eta-1}{1-\rho}}\right]^{\frac{1}{\eta-1}} = \xi_l^{\frac{\eta}{\eta-1}}\frac{A_l}{\phi}\left[q^{\eta-1}+\sigma_l^{\eta}\left[\tilde{I}_l\left(1-\kappa_l\right)^{-\rho}\right]^{\frac{\eta-1}{1-\rho}}\right]^{\frac{1}{\eta-1}}$$

$$\Longrightarrow q^{\eta-1}+\sigma_l^{\eta}\left[\tilde{I}_l\left(1-\kappa_l\right)^{-\rho}\right]^{\frac{\eta-1}{1-\rho}} = \left(\frac{\phi\chi}{A_l}\right)^{\eta-1}\xi_l^{\frac{\eta}{1-\eta}}$$

Using the expression for $\tilde{I}_l$ in (12) to obtain (20).

Deriving equation for $S\left(\chi; \frac{H}{L}, \frac{\phi}{A_l}\right)$: The labor market clearing condition for the high-skill and the low-skill labor together imply:

$$\frac{H_l}{L_l}\left(L-L_h\right)+\frac{H_h}{L_h}L_h = H,$$

thus the share of low-skill labor in the high-skill sector is:

$$l_h \equiv \frac{L_h}{L} = \frac{H/L - H_l/L_l}{H_h/L_h - H_l/L_l}, \tag{A.9}$$

simplify and use (13) to obtain the first equilibrium condition (21).

Deriving equation for $D\left(\chi; \hat{A}_{lh}, \frac{\phi}{A_l}\right)$: The goods market clearing conditions and the relative demand imply:

$$x = \frac{p_h C_h}{p_l C_l} = \frac{P_h Y_h}{P_l\left(Y_l - \phi K\right)} \Longrightarrow \frac{p_h Y_h}{p_l Y_l} = x\left(1-\frac{\phi K}{Y_l}\right), \tag{A.10}$$

where, using relative price (19), x is derived as

$$x = \hat{A}_{lh}^{1-\varepsilon}\left(\frac{\xi_h^{-\eta}J_h}{\xi_l^{-\eta}J_l}\right)^{\frac{1-\varepsilon}{\eta-1}}; \hat{A}_{lh} \equiv \frac{A_l}{A_h}\left(\frac{1-\psi}{\psi}\right)^{\frac{\varepsilon}{1-\varepsilon}}$$

and using the capital market clearing condition, K is derived as:

$$K = K_h + K_l = \frac{K_h}{L_h}L_h + \frac{K_l}{L_l}\left(L-L_h\right)$$

so the relative demand equation (A.10) can be written as

$$\frac{p_h Y_h}{x p_l Y_l} = 1 - \frac{\phi}{Y_l}\left(\frac{K_h}{L_h}L_h + \frac{K_l}{L_l}\left(L-L_h\right)\right),$$

given $\phi \equiv q_k/p_l$, rewrite it in terms of the low-skill income share J_j:

$$\frac{J_l}{xJ_h}\left(\frac{L_h}{L_l}\right) = 1 - \frac{q_k J_l}{q_l L_l}\left(\frac{K_h}{L_h}L_h + \frac{K_l}{L_l}\left(L-L_h\right)\right) = 1 - \frac{J_l}{L_l}\left(\frac{1-\beta_h}{J_h}L_h + \frac{1-\beta_l}{J_l}\left(L-L_h\right)\right),$$

where the equality follows from the definition of β_j. Finally (22) is derived from:

$$\frac{J_l}{xJ_h}\left(\frac{l_h}{1-l_h}\right) = 1 - \frac{J_l}{1-l_h}\left[\frac{1-\beta_h}{J_h}l_h + \frac{1-\beta_l}{J_l}\left(1-l_h\right)\right].$$

Value-added shares: The value-added share of the high-skill sector is:

$$v_h = \left[1+\frac{p_l Y_l}{p_h Y_h}\right]^{-1} = \left[1+\frac{p_l A_l F_l/L_l}{p_h F_h/L_h}\frac{L_l}{L_h}\right]^{-1}$$

Using relative prices (19) and (A.7), (24) is obtained from:

$$v_h = \left[1+\left(\frac{1-\lambda_h}{1-\lambda_l}\right)^{\frac{\eta}{\eta-1}}\left(\frac{J_l}{J_h}\right)^{\frac{1}{\eta-1}}\left(\frac{1-\lambda_l}{J_l}\right)^{\frac{\eta}{\eta-1}}\left(\frac{J_h}{1-\lambda_h}\right)^{\frac{\eta}{\eta-1}}\left(\frac{L_l}{L_h}\right)\right]^{-1}.$$

A.2.4. Calibration

This section explains how the weight of each input is calibrated to match the sectoral income share for period 0 and period T.

Normalization of $\phi/\mathbf{A_l}$: The initial $\frac{\phi}{A_l}$ can be normalized to 1. Note that

$$\tilde{I}_j = \left[1 + \frac{K_j}{\chi H_j}\right]^{-1} \Longrightarrow \frac{K_j}{\chi H_j} = \frac{1-\tilde{I}_j}{\tilde{I}_j},$$

which is independent of ϕ/A_l. Also using the definition of J:

$$J_j^{-1} = \left[1 + \frac{K_j}{\chi H_j}\right] q \frac{H_j}{L_j} + 1$$

so $\frac{H_j}{L_j}$ is independent of ϕ/A_l as well. It follows from (A.9) that l_h is independent of ϕ/A_l. Given H_j/L_j and K_j/H_j are independent of ϕ/A_1, so the allocation of all inputs is independent of ϕ/A_1. This shows that we can normalize $\phi/A_{l0} = 1$ as it does not affect input allocation across sectors. The value of ϕ_T/A_{lT} is then determined by the growth in the relative price of capital ϕ_T/ϕ_0 and the growth in the productivity of the low-skill sector A_{lT}/A_{l0}.

Calibration of $\kappa_\mathbf{l}, \xi_\mathbf{l}$: Given ϕ/A_l, equation (A.8) expresses χ as a function of ξ_l given data on q and J_l:. Substitute this into $\tilde{I}_l$ in (12) to solve for δ_l explicitly:

$$\delta_l = \left(\frac{1-\tilde{I}_l}{\tilde{I}_l}\chi^{1-\rho}\right)^{\frac{1}{\rho}},$$

which implies a value of $\kappa_l = \frac{\delta_l}{1+\delta_l}$ for any given level of ξ_l. Thus, the income share (14) provides an implicit function to solve for ξ_l given data on $\left(\tilde{I}_l, J_l\right)$:

$$J_l = \left[1 + q^{1-\eta}\sigma_l^\eta \left[\tilde{I}_l\left(1-\kappa_l\right)^{-\rho}\right]^{\frac{\eta-1}{1-\rho}}\right]^{-1},$$

This procedure pins down χ, ξ_l and κ_l. More explicitly:

$$\left(1-\kappa_l\right)^{-1} = 1 + \delta_l = 1 + \left[\frac{1-\tilde{I}_l}{\tilde{I}_l}\chi^{1-\rho}\right]^{\frac{1}{\rho}} = 1 + \left[\frac{1-\tilde{I}_l}{\tilde{I}_l}\left(\frac{q\phi}{A_l}J_l^{\frac{1}{1-\eta}}\xi_l^{\frac{\eta}{1-\eta}}\right)^{1-\rho}\right]^{\frac{1}{\rho}}$$

$$\Longrightarrow \sigma_l^\eta\left[\left(1-\kappa_l\right)^{-1}\right]^{\frac{\rho(\eta-1)}{1-\rho}} = \sigma_l^\eta\left[1 + \left(\frac{1-\tilde{I}_l}{\tilde{I}_l}\right)^{\frac{1}{\rho}}\left(qA_kJ_l^{\frac{1}{1-\eta}}\right)^{\frac{1-\rho}{\rho}}\xi_l^{\frac{\eta(1-\rho)}{(\eta-1)\rho}}\right]^{\frac{\rho(\eta-1)}{1-\rho}}$$

The implicit function is

$$f\left(\xi_l\right) = \left[1 + q^{1-\eta}\left[\left(\frac{1-\xi_l}{\xi_l}\right)^{\frac{\eta(1-\rho)}{\rho(\eta-1)}} + \left(\frac{1-\tilde{I}_l}{\tilde{I}_l}\right)^{\frac{1}{\rho}}\left(\frac{q\phi}{A_l}J_l^{\frac{1}{1-\eta}}\right)^{\frac{1-\rho}{\rho}}\left(1-\xi_l\right)^{\frac{\eta(1-\rho)}{(\eta-1)\rho}}\right]^{\frac{\rho(\eta-1)}{1-\rho}}\right]^{-1} - J_l,$$

where

$$f'\left(\xi_l\right) > 0, \lim_{\xi_l \to 1} f\left(\xi_l\right) = 1 - J_l > 0, \lim_{\xi_l \to 0} f\left(\xi_l\right) = -J_l < 0.$$

Thus, there is a unique solution for $\xi_l \in (0,1)$.

Calibration of κ_h, ξ_h: Using income share $\tilde{I}_h$ in (12):

$$\delta_h = \left[\frac{1-\tilde{I}_h}{\tilde{I}_h}\chi^{1-\rho}\right]^{\frac{1}{\rho}} \Longrightarrow \kappa_h = \frac{\delta_h}{1+\delta_h}.$$

Given $\tilde{I}_h$ and χ, κ_h is obtained. Using J_h in (14):

$$\sigma_h = \left[\frac{1-J_h}{J_h}q^{\eta-1}\left[\tilde{I}_h\left(1-\kappa_h\right)^{-\rho}\right]^{\frac{1-\eta}{1-\rho}}\right]^{\frac{1}{\eta}},$$

given $\kappa_h, \tilde{I}_h, J_h$ and q, ξ_h is obtained.

Data availability

Data will be made available on request.

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